PARALLEL LINES ASSUMPTION IN ORDINAL LOGISTIC REGRESSION AND ANALYSIS APPROACHES

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ABSTRACT
The aim of this article is to examine Proportional Odds Model (POM), Non- Proportional Odds Model (NPOM) and Partial Proportional Odds Model (PPOM) models and to determine the most suitable model according to the structure of data and assumptions. Data related to the variables which affect job satisfaction of media employees has been used in line with this aim, and Maximum Likelihood Estimator and Odds ratios of the examined models have been obtained. Model validity and comparison have been tested by Likelihood Ratio statistics. It has been concluded in the study that NPOM and PPOM are required to be preferred compared to POM and multinomial logit model when Parallel Lines Assumption is not hold.

Key Words: Parallel Lines Assumption, Proportional Odds Model (POM), Non- Proportional Odds Model (NPOM), Partial Proportional Odds Model (PPOM).

1. INTRODUCTION

Logit models are used to solve regressions with a single dependent variable and various independent variables. In logit models the natural logarithm of odds which belongs to ordinal dependent variable is expressed as a linear function of the independent variables, therefore logit model is a member of "generalized linear models" family and logit transformation (the natural logarithm of the independent variable's odds) is used as a link function.

Dependent variables which are analyzed in the majority of researches and applied studies are generally in categorical and ordinal structure. Ordinal Logit Models that consider the ordinal structure of the dependent variable are used in case where the dependent variable has at least 3 categories with these categories ordinally arranged, i.e. severe of disease (mild, moderate, severe) or the educational level (elementary, high, university) (Hosmer and Lemeshow, 2000).

There are various ordinal logit models to compare dependent variable categories. Easiest of these to apply or interpret are Cumulative Logit Models. Cumulative Logit Models are divided into 3 groups as Proportional Odds Model (POM), Non-Proportional Odds Model (NPOM) and Partial Proportional Odds Model (PPOM). Not like the Multinomial Logit Models, Cumulative Logit Models are work under the assumption of cumulative logit parallelity. But parallel lines assumption sometimes does not hold, in this case Proportional Odds Model gives incorrect results. Therefore models that consider ordinal structure and relax the assumption are suggested. NPOM and PPOM are recently used for this purpose.

1.1. Cumulative Logit Models

Various logit formats are used to compare dependent variable categories in ordinal logistic models. But cumulative logits are the easiest models when it comes to interpret or apply.

Like the other logit models, odds ratios are calculated to find cumulative probabilities in cumulative logit models. There are j – 1 ways to compare j categorized dependent variable Y. Equality shows odds ratio of dependent variable Y for $(Y \geq 1, Y < 1; Y \geq 2, Y < 2; \ldots Y \geq J-1, Y < J-1)$ (Kleinbaum and Klein, 2010).

$$P(Y \geq J) = \frac{P(Y \geq J)}{P(Y < J)} \quad [1.]$$

According to equality 1, dependent variable Y consists of J ordinal categories. Like the other models, one category is chosen as the reference category (generally the highest category). J – 1 cut-off points are estimated this way and the estimations give information for every consecutive category about cumulative probabilities. Probability of being in the chosen category or being in a lower category are taken into account together in cumulative probability (O'Connell, 2006).

Generally cumulative probability is calculated as the sum of the category probabilities as it is shown in Equality 2. (Agresti, 2002).

$$(Y \leq j/x) = \pi_1(x) + \pi_2(x) + \cdots + \pi_j(x) \qquad j = 1, \ldots, J \qquad [2.]$$

Cumulative logit model is derived from logit link function as shown in Equality 3.

$$logit[(Y \leq j|x)] = log\left[\frac{P(Y \leq j|x)}{1 - P(Y \leq j|x)}\right]$$

$$= log\left[\frac{\pi_1(x) + \pi_2(x) + \cdots + \pi_j(x)}{\pi_{j+1}(x) + \cdots + \pi_J(x)}\right] \quad j = 1, \ldots, J-1 [3.]$$

$(P\,(Y \leq j|x)/1 - P(Y \leq j|x))$ in Equality 3 are defined as cumulative odds for J. dependent variable category.

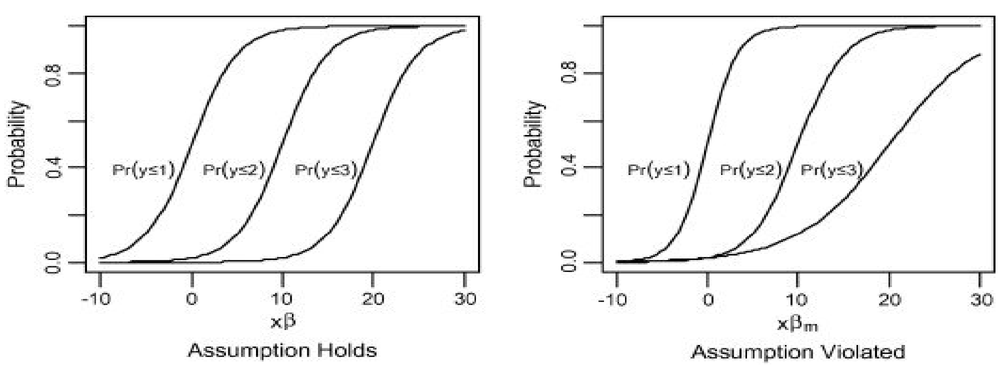
Cumulative logit models are divided into three main model group according to the parallelity assumption. These models are Proportional Odds Model proposed by McCullagh, Non-Proportional Odds Model proposed by Fu and Partial Proportional Odds Model proposed by Harrel (McCullagh, 1980; Fu, 1998; Peterson and Harrell, 1990).

1.1.1. Parallel Lines Assumption

In ordinal logistic regression models there is an important assumption which belongs to ordinal odds. According to this assumption parameters should not change for different categories. In other words, correlation between independent variable and dependent variable does not change for dependent variable's categories, also parameter estimations do not change for cut-off points. In an ordinal logit regression, when the assumption holds for j – 1 logit comparison in a J categorized variable, α_{J-1}cut-off points and j – 1βparameters are found. At this point ordinal logistic model differs from multinomial logistic regression (Kleinbaum and Klein, 2010).
In a way, this assumption states that the dependent variable's categories are parallel to each other. When the assumption does not hold, it means that there are no parallelity between categories. Graph 1 shows the conditions (Fullerton and Xu, 2012).
Likelihood Ratio Test, Wald Chi-Square test and the other related tests are used to test parallel lines assumption (Long, 1997; Agresti, 2002). In ordinal logit regression, these tests examine the equality of the different categories and decides whether the assumption holds or not. If the assumption does not hold, interpretations about results will be wrong, therefore in order to find correct results alternative models are used instead of ordinal logit regression models.
Equality 4 shows the hypothesis that tests whether $\beta_k$coefficients of independent variable are equal or not for every single category.

$$H_0 = \beta_{1j} = \beta_{2j} = \cdots = \beta_{(K-1)j} = \beta \qquad j = 1,2,\ldots,J[4.]$$

Graph 1. Conditions where the assumption holds and does not hold.



1.1.2. Proportional Odds Model (POM)

In case where the dependent variable is ordinal and parallel lines assumption holds, Proportional Odds Models are commonly used (Brant, 1990; Bender and Grouven, 1998).
Proportional Odds Model is defined by McCullagh(1980) for ordinal logistic regression. The model is based on cumulative distribution function.
Proportional odds models can be estimated as shown in equality 5 using cumulative probabilities (Kleinbaum and Ananth, 1997).

$$P(Y \leq y_j|x) = \left[\frac{exp(\alpha_j - x'\beta)}{1 + exp(\alpha_j - x'\beta)}\right] \qquad j = 1,2,\ldots,J-1[5.]$$

Where α_j is the unknown parameter's estimator and index j = 1, 2, … , j – 1 for $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_{j-1}$; β is the regression parameter vector of x and shown as $\beta = (\beta_1, \ldots, \beta_k)'$. The model can be transformed to linear form via calculating natural logarithms of the odds ratios. (Kleinbaum and Ananth, 1997).

$$\left(\alpha_j - x'\beta\right) \quad j = 1,2,\ldots,J-1 \quad [6.]$$

In proportional odds model, every single cumulative logit has its own threshold value.β coefficients of the equality are independent from dependent variable categories, which are shown as "j"(j = 1, 2, … , j – 1) . Thus β coefficients of the independent variable will be equal to each other in every cumulative logit model (Kleinbaum and Ananth, 1997; McCullagh and Nelder, 1989).

1.1.3. Non-Proportional Odds Model (NPOM)

In case where the dependent variable is ordinal, parallel lines assumption sometimes does not hold. In this case multinomial analyze technique can be used. But multinomial analysis disregards the ordinal structure of the dependent variable and assumes it as a nominal variable. Therefore, using multinomial analysis when the assumption does not hold causes information loss. At this point a new model which can relax the parallel lines assumption and considers the ordinal structure is suggested.

Proposed by Fu (1998), "Non-Proportional Odds Model" also uses cumulative logit to build the model and does not hold the parallel lines assumption. Therefore, the effect of the independent variable's odds to dependent variable will not be equal and β coefficients will be different for every single category of the dependent variable which is shown as m (m = 1, 2, …, M – 1) (Fu, 1998). The most significant difference between POM and NPOM is that NPOM has different parameters for every single category of the dependent variable.

Equality 7 shows the cumulative probability equation for Non-proportional Odds Model (Generalized Ordinal Logit Model) (Fullerton and Xu, 2012; Williams, 2006).

$$P(Y \leq y_m|x) = \left[\frac{exp(\tau_m - x'\beta_m)}{1 + exp(\tau_m - x'\beta_m)}\right] \qquad m = 1,2,\ldots,M-1[7]$$

Where τ_m isunknown parameter's estimator and has $M-1$ estimators and $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_{m-1}$ ordered treshold, vectors of the regression coefficients are $\beta(\beta = (\beta_{m1}, \ldots, \beta_{mk})')$. The model is transformed via calculating the natural logarithm of the odds ratio and it will take the linear form shown in Equality 8 (Fullerton and Xu, 2012).

$$(\tau_m - x'\beta_m) \quad m = 1,2,\ldots,M-1[8.]$$

Every single cumulative logit has its own threshold value. When showing categories as$m = 1,2,\ldots,M-1$, β_m coefficients have different values for every single category of dependent variable.

1.1.4. Partial Proportional Odds Model (PPOM)

Suggested by Peterson and Harrell (1990), Partial Proportional Odds Model can be used when parallel lines assumption holds or not. Partial Proportional Odds Model bears the same characteristics with both Proportional Odds Model and Non-Proportional Odds Model (Peterson and Harrell, 1990; Ananth and Kleinbaum, 1997).

PPOM is defined in two forms, constrained odds model and unconstrained odds model, by Peterson and Harrell (1990).

1.1.4.1.Unconstrained Partial Proportional Odds Model (UPPOM)

Two different coefficient set are estimated in UPPOM; first set holds the parallel lines assumption and the second set does not hold the parallel lines assumption.

Equality 9 shows the general form of the model (Ananth and Kleinbaum, 1997).

$$P\left(Y \leq y_j|x\right) = \frac{exp\left(-\alpha_j - x'\beta - t'\gamma_j\right)}{1 + exp\left(-\alpha_j - x'\beta - t'\gamma_j\right)} \quad j = 1,2, \ldots, k[9.]$$

Whereα_j is unknown parameter's estimator, x is the (p x 1) scaled vector that points the parameters which are holding the parallel lines assumption and lastly the t is the (q x 1) scaled vector that points the parameters which are not holding the parallel lines assumption. If all of the parameters have non-proportional characteristics, model becomes NPOM. In case, where q is smaller than p, only one subset of parameters will be non-proportional, therefore model becomes PPOM. Here in this equationγ_j is associated with the coefficients and defines increment changes in logit for non-proportional variables. If γ_j is equal to 0, parallel lines assumption holds and model becomes POM (Peterson and Harrel, 1990; Ananth and Kleinbaum,1997). The model is transformed via calculating the natural logarithm of the odds ratio and it will take the linear form shown in Equality 10 (Jones and Westerland, 2006).

$$\left(-\alpha_j - x'\beta - t'\gamma_j\right) \quad j = 1,2, \ldots, k[10.]$$

1.1.4.2.Constrained Partial Proportional Odds Model (CPPOM)

In non-proportional odds model, constraints are defined for a group of non-proportional variables. The Model becomes constrained when the coefficients at the changing break points are multiplied with a predefined constant scalar. As there will be parallelity between constrained coefficients of variable, CPPOM will need fewer parameters compared to unconstrained PPOM and NPOM (Ananth and Kleinbaum, 1997). General form of the model proposed by Peterson and Harrell (1990) is shown in equality 11:

$$P\left(Y \leq y_j|x\right) = \frac{exp\left(-\alpha_j - x'\beta - t'\gamma\Gamma_j\right)}{1 + exp\left(-\alpha_j - x'\beta - t'\gamma\Gamma_j\right)} \quad j = 1,2, \ldots, k[11.]$$

Where α_j is unknown parameter's estimator, x is the $(p \times 1)$scaled vector that points the parameters which are holding the parallel lines assumption and lastly the t is the $(q \times 1)$ scaled vector that points the parameters which are not holding the parallel lines assumption. Γ_j is the predefined constant scalar and γ vector is q scaled and does not depend on J. Cumulative logit is calculated multiplying γ by Γ_j,even thoughγ independent from J. The model is transformed via calculating the natural logarithm of the odds ratio and it will take the linear form shown in Equality 12 (Peterson and Harrel, 1990; Ananth and Kleinbaum, 1997).

$$\left(-\alpha_j - x'\beta - t'\gamma\Gamma_j\right) \quad j = 1,2, \ldots, k[12.]$$

1.2.Multinomial Logistic Regression Analysis

In multinominal logistic regression, dependent variable has more than two nominal categories which do not have an ordinal structure. Also this dependent variable is multinominally distributed. Shortly, it is an expanded form of binary logistic regression model for J categories. A multinominal logistic model with J categories must have $J - 1$ logistic regressions. (Liao, 1994).

In a multinominal logistic regression model, dependent variable's probablity of being in category j$\pi_j = P(Y = j)$ is shown in Equality 13 (Liao, 1994).

$$\pi_j = \frac{exp\left(\sum_{k=1}^{K} \beta_{jk} x_k\right)}{1 + \sum_{j=1}^{J-1}\left(\sum_{k=1}^{K} \beta_{jk} x_k\right)} \quad j = 1,2, \ldots, J - 1[13.]$$

and in Equality 14.

$$\pi_j = \frac{1}{1 + \sum_{j=1}^{J-1} exp - \left(\sum_{k=1}^{K} \beta_{jk} x_k\right)} [14.]$$

2. TOOLS AND THE METHOD

The main purpose of this study is examining ordinal and multinomial models when dependent variable has more than two categories and comparing them to choose the best model via goodness of fit indicators, considering data structure and assumptions where the parallel lines assumption does not hold. For this purpose, factors which have effects on job satisfaction of media workers used as the data and odds and maximum likelihood estimators of related models are collected. Likelihood ratio test is used to test effectiveness of these models. Lastly deviation measure, pseudo R^2 values, Akaike Information Criteria and Bayesian Information Criteria are used to indicate goodness of fit.

Data about factors which have effects on job satisfaction of media workers are collected via detailed surveys filled by 234 random journalist chosen from Ankara offices of several papers, magazines and news agencies. Data collected from surveys analyzed and interpreted with the help of STATA 10(Long and Freese, 2006; Hamilton, 2009).

In this study job satisfaction level is used as the ordinal dependent variable. Job satisfaction level is in an ordinal form and ordered as 1: Low, 2: Normal, 3: High. Sex, age group, educational status, marital status, seniority, income (monthly), employee's income group, employee's dissidence from the company individual works in and the type of the media organ are the independent variables which should have an effect on job satisfaction level. Categories of independent variables can be seen on Table 1.

Some of independent variables given in Table 1 are chosen to use in the logistic regression analysis and relationship between the dependent variable and independent variables is investigated with Chi-Square analysis. (According to Chi-Square results, independent variables income (monthly), employee's income group, employee's dissidence from the company individual works in and type of the media organ are found significant.

Collinear ties between chosen dependent variables are investigated. Inverse diagonals of the variance-covariance matrix (VIF values) are calculated and used to determine collinearity problem. Calculated VIF values are smaller than 5, therefore there is no collinearity between independent variables (Allison, 2012).

Effects of income (monthly), employee's income group, employee's dissidence from the company individual works in and type of the media organ are investigated with ordinal and multinomial logit models in detail. Parallel lines assumption is tested in ordinal logit models and where the assumption does not hold, alternative logit models are examined with likelihood ratio test. Validity of models are tested with parallel lines assumption and likelihood ratio test then between ordinal logit models; proportional odds model, non-proportional odds model and multinomial logit model; best model which fits the data is chosen with the help of goodness of fit indicators.

Table 1. Independent variables and levels

Independent Variable	Independent variable levels		
X1: Sex	1: Women	2: Men	
X2: Age group	1: Under 35	2: Between 36-45	
X3:Educational status	1: high school	2: above undergraduate	
X4: Marital status	1: married	2: single	3: widow
X5: Seniority	1: Above 10 years	2: Between11-20 years	3: Above 21 y.
X6: Income	1: Under 2000TL	2: Above 2000TL	
X7: Employee's income group	1: Poor	2: Middle Class	3: Rich
X8: Employee's dissidence from the company individual works in	1: I agree with the political attitudes/publishing policy of my company	2: I don't agree with the political attitudes/publishing policy of my company but it is not a problem.	3: I don't agree with the political attitudes/publishing policy of my company, therefore I can't work as I wish.
X9: Media Organ	1: Public	2: Private sector	

3. FINDINGS

Detailed statistics obtained from mentioned models can be found in following tables.

Table 2. Shows the frequency distribution of the dependent variable job satisfaction level.

Results in Table 3 is calculated using proportional odds model.

Validity of the proportional odds model tested by likelihood ratio test and found significant (p = 0.0157). Although it is significant, parallel lines assumption must be tested. Equality of parameters from different categories should be tested to examine if the assumption holds or not. Equality 15 shows the hypothesis for our analysis.

$$H_0 = \beta_{1k}=\beta_{2k} = \cdots = \beta_{(j-1)k} \quad j = 1,2, \ldots, J-1 \; [15.]$$

Parameters of the independent variable k are tested with the ratio test to see if they are equal or not. For $\chi^2 = 20,83; p = 0,0003$ values null hypothesis is rejected, therefore parallel lines assumption does not hold.Parallel lines assumption also tested with Wald test proposed by Brant. Table 4 shows the results.

As it can be seen on Table 4 parallel lines assumption does not hold according to Brant's Wald Test too. Wald test gives information about Independent variable or variables that break parallel lines assumption. Parallel lines assumption does not hold even only one variable breaks the assumption. According to Table 4, X8 and X9 (employee's dissidence from the company individual works in and type of the media organ) break the assumption of parallel lines, therefore null hypothesis is rejected. Table 5 shows goodness of fit indicators of the Proportional Odds Model.

Results of the non-proportional model odds model can be seen in Table 6.

Validity of the non-proportional odds model tested by likelihood ratio test and found significant (χ^2= 33,58 and$p = 0,000$). It can be seen that β coefficients take different values for every single category in the model. Therefore odds ratios are changing for every single category. For example coefficient of the income variable does not hold parallel lines assumption, thus it takes different values for every single category of the dependent variable. The same condition applies to other variables too.Table 7 shows goodness of fit indicators of the non-proportional odds model.

Results of the partial proportional model can be seen in Table 8.

In this model parallel lines assumption is tested in %5 significance level by assigning constraints to coefficients. For employee's dissidence from the company individual works in and type of the media organ variables this constraints don't hold parallel lines assumption for %5 significance level $(p = 0{,}000 <$ *0,05;p=0,011<0,05.*

Tested by Brant's Wald Test, coefficients hold the parallel lines assumption (χ^2=1,72 and $p = 0{,}422 > 0{,}05$) and partial proportional lines model is found significant (χ^2=31,70 and $p = 0{,}000$).As it can be seen on Table 7. Coefficients of income (monthly) and employee's income group variables do not change for dependent variable's different categories, therefore it can be said that the assumption holds for these variables. Table9 shows goodness of fit indicators of the Partial Proportional Odds Model results of the Partial Constrained Partial Proportional Odds Model can be seen in Table 10.

Tested by Brant's Wald Test, coefficients hold the parallel lines assumption and constrained partial proportional lines model is found significant (χ^2 =12,24 ; $p = 0{,}0157 < 0{,}05$). Shortly, instead of estimating different parameters for every variable, only one parameter is estimated for all variables.

Table 11 shows goodness of fit indicators for Constrained Partial Proportional Odds model.

Results of the Unconstrained Partial Constrained Partial Proportional Odds Model can be seen in Table 12.

As it can be seen on coefficient values at Table 12, there are different coefficients for every category. Model is significant with χ^2=33,58 ; $p = 0{,}000 < 0{,}05$ values. When looking at comparisons, odds values of dissidence and media organ variables are statistically important for comparison I. Table 13. shows goodness of fit indicators of the unconstrained partial proportional odds model.

Results of the multinomial logit model can be seen in Table 14.

Multinomial logit model is significant (χ^2=30,12 ; $p = 0{,}000 < 0{,}05$) when tested with likelihood ratio test. When looking at comparisons, odds values of dissidence and media organ variables are statistically important for comparison I. Table 15 shows goodness of fit indicators of the multinomial logit model.

Table 2. Frequancy distribution of the job satisfaction level

Job Satisfaction Level	Freq.	Percent	Valid percent	Total percent
14-32 point (Low)	18	7,7	7,8	7,8
33-52 point (Normal)	125	53,4	53,9	61,6
53-70 point (High)	89	38,0	38,4	100,0
Total	232	99,1	100,0	
Missing Observations	2	,9		
Total	234	100,0		

Table 3. Coefficients, standart errors, odds ratio estimates and p values of the Proportional Odds Model

Job Satisfaction Level	Variable	Coef. $(\hat{\beta})$	Standard Error	Odds Ratio $(e^{\hat{\beta}})$	p value
Low category vs. Normal and High category (Comparison 1)	Threshold 1	-0,562	1,205	---	---
Low and normal category vs. High category (Comparison 2)	Threshold 2	2,511	1,218	---	---
	Income	0,476	0,274	1,609	0,083
	Income group	0,971	0,475	2,640	0,041*
	Dissidence	-0,162	0,176	0,850	0,359
	Media Organ	-0,244	0,279	0,783	0,381

(*p < 0,05)

Table 4. Results of Wald Test Proposed by Brant

Variable	χ^2	p	s.d
All	15,53	0,004*	4
X6	1,01	0,167	1
X7	0,02	0,737	1
X8	10,65	0,001*	1
X9	4,09	0,043*	1

($*p < 0{,}05$)

Table 5. Goodness of fit indicators of the Proportional Odds Model

Goodness of Fit Indicators	
Mac Fadden R^2	0,0303
Deviation	391,5696
AIC	403,5996
BIC	424,0395

Table 6. Coefficients, standart errors, odds ratio estimates and p values of the Non-Proportional Odds Model

Job satisfaction level	Variable	Coef. ($\hat{\beta}$)	Standard Error	Odds Ratio ($e^{\hat{\beta}}$)	p value
Low category vs. Normal and High category (Comparison 1)	Threshold 1	5,198	2,439	---	---
	Income	1,141	0,613	3,132	0,063
	Income group	0,788	0,793	2,199	0,321
	Dissidence	-1,294	0,358	0,273	0,000*
	Media Organ	-1,857	0,791	0,156	0,019*
Low and normal category vs. High category (Comparison 2)	Threshold 2	-3,046	1,379	---	---
	Income	0,334	0,287	1,397	0,245
	Income group	0,981	0,552	2,668	0,075
	Dissidence	0,073	0,181	1,076	0,686
	Media Organ	-0,032	0,293	0,967	0,911

($*p < 0{,}05$)

Table 7. Goodness of fit indicators of the Non-Proportional Odds Model.

Goodness of Fit Indicators	
Mac Fadden R^2	0,0832
Deviation	370,2256
AIC	390,2256
BIC	424,3421

Table 8. Coefficients, standart errors, odds ratio estimates and p values of the Partial Proportional Odds model.

Job satisfaction level	Variable	Coef. $(\hat{\beta})$	Standard Error	Odds Ratio $(e^{\hat{\beta}})$	p value
Low category vs. Normal and High category (Comparison 1)	Threshold	5,921	2,045	---	---
	Income	0,439	0,277	1,551	0,113
	Income group	0,958	0,479	2,607	0,046*
	Dissidence	-1,233	0,348	0,291	0,000*
	Media Organ	-1,996	0,781	0,135	0,011*
Low and normal category vs. High category (Comparison 2)	Threshold 2	-3,139	1,243	---	---
	Income	0,439	0,277	1,551	0,113
	Income group	0,958	0,479	2,607	0,046*
	Dissidence	0,056	0,181	1,057	0,758
	Media Organ	-0,028	0,293	0,971	0,922

(*p < 0,05)

Table 9. Goodness of fit indicators of Partial Proportional Odds Model

Goodness of Fit Indicators	
Mac Fadden R^2	0,0785
Deviation	372,1018
AIC	388,1018
BIC	415,395

Table 10. Coefficients, standart errors, odds ratio estimates and p values of the Constrained Partial Proportional Odds Model

Job satisfaction level	Variable	Coef $(\hat{\beta})$	Standard Error	Odds Ratio $(e^{\hat{\beta}})$	p value
Low category vs. Normal and High category (Comparison 1)	Threshold 1	0,562	1,205	---	---
	Income	0,476	0,274	1,609	0,083
	Income group	0,971	0,475	2,640	0,041*
	Dissidence	-0,162	0,176	0,850	0,359
	Media Organ	-0,244	0,279	0,783	0,381
Low and normal category vs. High category (Comparison 2)	Threshold 2	-2,802	1,218	---	---
	Income	0,482	0,274	1,609	0,083
	Income group	1,028	0,475	2,640	0,041*
	Dissidence	-0,159	0,176	0,850	0,359
	Media Organ	-0,126	0,279	0,783	0,381

(*p < 0,05)

Table 11. Goodness of fit indicators for Constrained Partial Proportional Odds model

Goodness of Fit Indicators	
Mac Fadden R^2	0,0303
Deviation	391,5696
AIC	403,5996
BIC	424,0395

Table 12. Coefficients, standart errors, odds ratio estimates and p values of the Unconstrained Partial Proportional Odds Model.

Job satisfaction level	Variable	Coef ($\hat{\beta}$)	Standard Error	Odds Ratio ($e^{\hat{\beta}}$)	p value
Low category vs. Normal and High category (Comparison 1)	Threshold	5,198	2,439	---	---
	Income	1,141	0,613	3,132	0,063
	Income group	0,788	0,793	2,199	0,321
	Dissidence	-1,294	0,358	0,273	0,000*
	Media Organ	-1,857	0,791	0,156	0,019*
Low and normal category vs. High category (Comparison 2)	Threshold 2	-3,046	1,379	---	---
	Income	0,334	0,287	1,397	0,245
	Income group	0,981	0,552	2,668	0,075
	Dissidence	-0,073	0,181	1,076	0,686
	Media Orhan	-0,032	0,293	0,967	0,911

(*$p < 0,05$)

Table 13. Goodness of fit indicators of the Unconstrained Partial Proportional Odds Model

Goodness of Fit Indicators	
Mac Fadden R^2	0,0832
Deviation	370,2256
AIC	390,2256
BIC	424,3421

Table 14. Coefficients, standart errors, odds ratio estimates and p values of the Multinominal Logit Model

Job satisfaction level	Variable	Coef. ($\hat{\beta}$)	Standard Error	Odds Ratio ($e^{\hat{\beta}}$)	p value
Low category vs. Normal category (Comparison 1)	Threshold 1	4,844	2,481	---	---
	Income	0,879	0,636	2,410	0,167
	Income group	0,609	0,800	1,839	0,446
	Dissidence	-1,166	0,361	0,311	0,001*
	Media Organ	-1,704	0,805	0,181	0,034*
Low category vs. High category (Comparison 2)	Threshold 2	1,503	2,648	---	---
	Income	1,113	0,644	3,045	0,084
	Income group	1,609	0,899	5,001	0,073
	Dissidence	-0,932	0,369	0,393	0,012*
	Media Orhan	-1,572	0,813	0,207	0,053

*($p < 0,05$)

Table 15. Goodness of fit indicators of the Multinominal Logit Model.

Goodness of Fit Indicators	
Mac Fadden R^2	0,0746
Deviation	373,6878
AIC	393,6879
BIC	427,8044

4. DISCUSSION AND CONCLUSION

In the study proportional odds model, non-proportional odds model and partial proportional odds model are found significant ($p = 0{,}0157; p = 0{,}000; p = 0{,}000$). Although it is significant, proportional odds model does not hold parallel lines assumption (LR Test; $p = 0{,}0003$; Brant's Wald Test; $p = 0{,}004$), therefore odds values are different for every cumulative category. Odds ratios inTable 3. are calculated with a false assumption, thus results do not show true condition of the data.

In NPOM, cumulative odds values are calculated without considering parallel lines assumption, therefore real slope estimates of odds values are calculated for both categories. In PPOM, parallel lines assumption is tested by defining constraints to variables. Parallel lines assumption does not hold for dissidence and media organ variables under %5 significance level ($p = 0{,}000 < 0{,}05; p = 0{,}011 < 0{,}05$) when all the variables are tested. Assumption holds for other variables ($p = 0{,}422 > 0{,}05$). Odds ratio estimates of models studied in this paper are compared in Table 16 and 17.

As it can be seen in Table 16, in NPOM income variable's odds ratios are estimated as 3,132 and 1,397. Odds of the income variable is equal for every single category (1,609) in POM. Income variable is tested for parallel lines assumption and shown in Comparison 1 and 2 (3,132; 1,397). When comparisons are statistically equal parallel lines assumption holds. But when parallel lines assumption does not hold, odds values in Comparison 1 and 2 will be different and they will be wrongly treated. For example dissidence variable does not hold the assumption and odds ratios estimated with NPOM are 0,273 and 1,076. At the other hand odds ratios of dissidence variable are equal for every single category of dependent variable (0,850) in POM. In this case it is necessary to use NPOM to determine the real situation and to make healthy interpretations.

In case the parallel line assumption does not hold, PPOM can be used instead of POM. PPOM is used when the assumption does not hold for some of the variables. In this regard, the model bears the characteristics of POM and NPOM. Dissidence and media organ variables were found as the variables which break the assumption in Findings section (Table 4). Where dissidence and media organ variables don't hold the assumption and other two hold, PPOM is applied to data and odds coefficients shown in Comparison 1 and 2 (Table 16) are estimated. Odds coefficients of income (1,551) and income group (2,607) variables are the same for every single category of the dependent variable. But dissidence variable does not hold the assumption and its odd ratios are 0,291 for Comparison 1 and 1,057 for Comparison 2. Similarly media organ variable's odds ratios are 0,135 for Comparison 1 and 0,971 for Comparison 2.

Constrained and unconstrained PPOM models which are invented in recent years are used frequently and these models are found significant (p = 0,015; p = 0,000). A common coefficient is estimated in constrained PPOM by giving constraints to coefficients, thus the assumption holds. When PPOM is applied to data, odds ratios in Comparison 1 and 2 will be equal. For example odds ratios of income variable (1,609) are the same in Comparison 1 and 2 and odds ratios of media organ variable (0,783) are the same in Comparison 1 and 2 (Table 16). The same condition applies to other variables too. Shortly, only one parameter is estimated instead of estimating different parameters for every single category and this way model is transformed to

POM. As it can be seen in Table 16 and 17, odds ratios estimated with POM and constrained PPOM are equal to each other.

Unconstrained PPOM reflects the non-proportional characteristics of PPOM. When applied to data, for every single variable different odds ratios are estimated (shown in Comparison 1 and 2). For example, odds ratio of media organ is 0,156 in Comparison 1 and 0,967 in Comparison 2. The same condition applies to other variables too. As it can be seen in Table 16odds ratios estimated with NPOM and unconstrained PPOM are equal to each other. Shortly, in a manner of speaking unconstrained PPOM is transformed to NPOM.

Where there are more than 2 dependent variables and the assumption does not hold multinomial logit model can be used instead of POM. The model is found significant (p = 0,000) when our multinomial logit model is applied to data. According to results, media organ and dissidence variables are found significant in the comparison between low category and normal category. When estimated with NPOM, Dissidence and media organ variables are found significant when low category is compared to normal category or high category. But although significant variables are exactly the same in both models, coefficients are completely different. In the multinomial logit model, dissidence variable's odds ratio estimated from comparison between low and normal categories is 0,311 while in NPOM it is 0,273 (low to normal and high). While odds ratio of the media organ variable is calculated 0,181 in multinomial logit regression, it is calculated 0,156 in NPOM. Even though significant variables are the same in PPOM and multinomial logit model, their coefficients are different. While odds ratio of dissidence and media organ variables are calculated 0,291 and 0,181 in PPOM, they are calculated 0,311 and 0,156 in multinomial logit model. Coefficients differ between PPOM and multinomial model as multinomial logit model does not include the ordinal structure and the high category group to estimations.

Table 18. shows the comparisons between ordinal logit models in terms of goodness of fit indicators.

NPOM is better than POM for the data compared in terms of goodness of fit indicators shown in Table 17. (Mac Fadden R^2, Deviation, AIC). Higher Mac Fadden R^2 values; lower deviation, AIC and BIC values are better and they show which model is better than the other one in terms of goodness of fit indicators. Also proportional odds model does not hold Parallel Lines "Assumption and Mac Fadden R^2, Deviation, AIC indicators give better results for NPOM. These results prove that, NPOM is a better model in terms of goodness of fit indicators when the assumption does not hold.

It can be said that PPOM fits the data better than POM in terms of goodness of fit indicators (Mac Fadden R^2, Deviation, AIC, BIC). In PPOM 2 of the variables holds the assumption and 2 of them are not. Therefore PPOM is an alternative to POM where the parallel lines assumption breaks down completely.

When PPOM compared to NPOM, Mac Fadden $R^2$and deviation values are better for NPOM and AIC and BIC values are better for PPOM. Therefore it is hard to say which model is better. But variables that holds the assumption have only one odds coefficient in PPOM and due to this fact it is useful when it comes to make statistical interpretations. Goodness of fit indicators show that Unconstrained PPOM and NPOM has the same indicators, while Constrained PPOM and POM has the same indicators.

When the parallel lines assumption does not hold, it has to be decided if it is better to use multinominal logit model or ordinal logit models and it has to be decided which one of them will be better for the data. Table 19. shows which model is better in terms of goodness of fit indicators.

Ordinal logit models fit better to the data in terms of goodness of fit indicators (Mac Fadden R^2, Deviation, AIC, BIC) shown in Table 18. But when the parallel lines assumption does not hold, it can't be decided whether multinomial logit model is better than the ordinal logit models or not. Because comparisons of odds estimations are different for multinomial logit model and ordinal logit models. References and categories used to create logits are cumulative in ordinal logit models. Therefore odds ratios of proportional, non-proportional and partial proportional models are cumulative. In multinomial logit models categories are nominally compared to a reference category and ordinal structure of the dependent variable is ignored. Therefore when the parallel lines assumption does not hold, NPOM and PPOM are better. Because they take all the information dependent variable provides into account and they don't ignore ordinal structure.

Table 16. Comparison of odds ratio estimates of POM, NPOM, UPPOM.

Proportional Odds Model					
	Variables	OR $(e^{\hat{\beta}})$			
Comparison 1 Comparison 2	Income	1,609			
	Income G.	2,640*			
	Dissidence	0,850			
	Media Organ	0,783			
Non-Proportional Odds MOdel			Unconstrained Partial Proportional Odds Model		
Job satisfaction level	Variables	OR $(e^{\hat{\beta}})$	Job satisfaction l.	Variables	OR$(e^{\hat{\beta}})$
Comparison 1	Income	3,132	Comparison 1	Income	3,132
	Income G.	2,199		Income G.	2,199
	Dissidence	0,273*		Dissidence	0,273*
	Media Organ	0,156*		Media Organ	0,156*
Comparison 2	Income	1,397	Comparison 2	Income	1,397
	Income G.	2,668		Income G.	2,668
	Dissidence	1,076		Dissidence	1,076
	Media Organ	0,967		Media Organ	0,967

Table 17. Comparison of odds ratio estimates of PPOM, CPPOM, UPPOM and Multinominal Logit Models

Partial Proportional Odds Model			Constrained Partial Proportional Odds Model		
Job satisfaction level	Variables	OR $(e^{\hat{\beta}})$	Job satisfaction l.	Variables	OR$(e^{\hat{\beta}})$
Comparison 1	Income	1,551	Comparison 1	Income	1,609
	Income G.	2,607*		Income G.	2,640*
	Dissidence	0,291*		Dissidence	0,850
	Media Organ	0,135*		Media Organ	0,783
Comparison 2	Income	1,551	Comparison 2	Income	1,609
	Income G.	2,607*		Income G.	2,640*
	Dissidence	1,057		Dissidence	0,850
	Media Organ	0,971		Media Organ	0,783
Multinominal Logistic Regression					
Job satisfaction level	Variables	OR $(e^{\hat{\beta}})$	Job satisfaction l.	Variables	OR$(e^{\hat{\beta}})$
Comparison 1	Income	2,410	Comparison 2	Income	0,084
	Income G.	1,839		Income G.	0,073
	Dissidence	0,311*		Dissidence	0,012*
	Media Organ	0,181*		Media Organ	0,053

Table 18. Comparisons between POM, NPOM, PPOM, CPPOM and UCPPOM in terms of goodness of fit indicators.

Goodness of fit	Models				
	POM	NPOM	PPOM	Const. PPOM	Unconst. PPOM
Mac Fadden R^2	0,0303	0,0832	0,0785	0,0303	0,0832
Deviation	391,5696	370,2256	372,1018	391,5696	370,2256
AIC	403,5996	390,2256	388,1018	403,5996	390,2256
BIC	424,0395	424,3421	415,395	424,0395	424,3421

Table 19. Comparisons between NPOM, PPOM and Multinominal Logit Model in terms of goodness of fit indicators.

Goodness of fit indicators	Models		
	NPOM	PPOM	Multinominal
Mac Fadden R^2	0,0832	0,0785	0,0746
Deviation	370,2256	372,1018	373,6878
AIC	390,2256	388,1018	393,6879
BIC	424,3421	415,395	427,8044

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